

1.)  $y = x^3 + 7x^2 + 12x$   
 $y = x(x^2 + 7x + 12)$   
 $y = x(x+3)(x+4)$   
 $0 = x(x+3)(x+4)$   
 $x=0 \quad x=-3 \quad x=-4$

4.)  $y = a(x+2)(x-b)$   $a$  &  $b$   
 $0 = a(x+2)(x-b)$   $a$  &  $b$   
 are constants  
 $x=-2 \quad x=b$

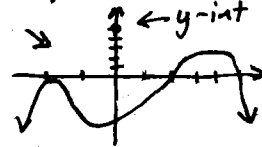
5.) Zeros at  $x=-2, 0, 3$   
 $x(x+2)(x-3)$   
 Double zero at  $x=-2$   
 as it bounces off and  
 doesn't cross  $x$ -axis.  
 $h(x) = x(x+2)^2(x-3)$

6.) Zeros at  $-2, 0, 2, 4$   
 $x(x+2)(x-2)(x-4)$   
 $g(x) = x(x+2)(x-2)(x-4)$   
 No double zeros

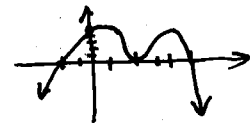
7.) Zeros at  $-2, 2$   
 $(x+2)(x-2)$   
 Triple zero at  $x=2$   
 since graph is flatter  
 or lingers there.  
 $f(x) = (x-2)^3(x+2)$

9.) Find possible formula for polynomial with zeros at  $x=-2, 2, 5$ .  
 A  $y$ -intercept of  $y=5$  and long-run behavior  
 $y \rightarrow -\infty$  as  $x \rightarrow \pm\infty$

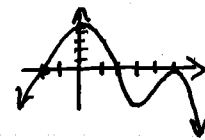
Must be even-powered degree, since  $y \rightarrow -\infty$  as  
 $x \rightarrow \pm\infty$ . So if it is 4<sup>th</sup> degree, it must have a  
 double-zero somewhere. Since the  $y$ -intercept is at  
 $(0, 5)$ , the double-zero can't be at  $x=-2$ .  
 Therefore, the double zero could be  
 at  $x=2$



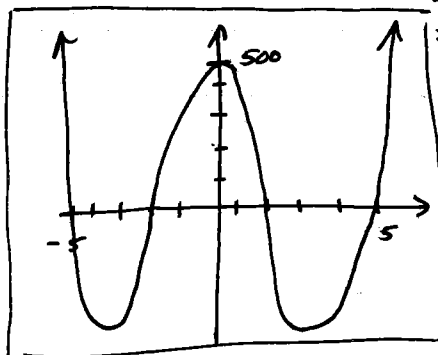
$y = k(x+2)(x-2)^2(x-5)$   
 $5 = k(0+2)(0-2)^2(0-5)$   
 $5 = -40k$   
 $-\frac{1}{8} = k$   
 $y = -\frac{1}{8}(x+2)(x-2)^2(x-5)$



or at  $x=5$   
 $y = k(x+2)(x-2)(x-5)^2$   
 $5 = k(0+2)(0-2)(0-5)^2$   
 $5 = -100k$   
 $-\frac{1}{20} = k$   
 $y = -\frac{1}{20}(x+2)(x-2)(x-5)^2$



10.)  $f(x) = -5(x^2-4)(25-x^2)$   
 $f(x) = -5(x+2)(x-2)(5+x)(5-x)$   
 $x$ -intercepts:  $x = -2, 2, 5, -5$   
 $f(0) = -5(0^2-4)(25-0^2) = -5(-4)(25) = 500$  ←  $y$ -intercept  
 Opens upward because:  $-5(x^2-4)(25-x^2)$   
 $= (-5x^2+20)(25-x^2)$   
 $= -125x^2 + 5x^4 + 500 - 20x^2$   
 $= 5x^4 - 145x^2 + 500$



Do not worry  
 about "k"  
 value in  
 these problems.

14.)  $p(x) = x^4 + 10x^3 - 68x^2 + 102x - 45$   
 Zeros at  $x = -15$   $x = 1$  (bounce)  
 and  $x = 3$

so  $p(x) = K(x+15)(x-1)^2(x-3)$   
 Since  $a = 1$  in non-factored,  $K = 1$

$$p(x) = (x+15)(x-1)^2(x-3)$$

15.) 4<sup>th</sup> degree opens downward, so lead coefficient must be negative with y-intercept of -6.

$$y = -\frac{1}{2}(x+2)(x+1)(x-2)(x-3)$$

$$y = -\frac{1}{2}(0+2)(0+1)(0-2)(0-3)$$

$$y = -\frac{1}{2}(12) = -6$$

19.) 3<sup>rd</sup> degree  $f(-3) = 0$   $f(1) = 0$   
 $f(4) = 0$   $f(2) = 5$

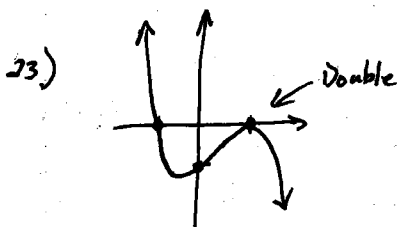
$$f(x) = K(x+3)(x-1)(x-4)$$

$$5 = K(2+3)(2-1)(2-4)$$

$$5 = -10K$$

$$-\frac{1}{2} = K$$

$$f(x) = -\frac{1}{2}(x+3)(x-1)(x-4)$$



Zeros  $x = -1$   $x = 1$

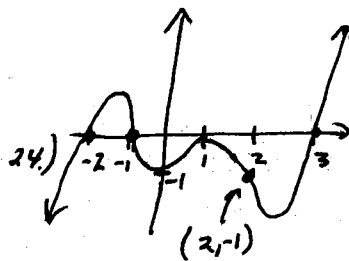
$$f(x) = K(x+1)(x-1)^2$$

y-int  $\rightarrow -1 = K(0+1)(0-1)^2$

$$-1 = K(1)$$

$$-1 = K$$

$$f(x) = -(x+1)(x-1)^2$$



Zeros:  $x = -2$   $x = -1$   
 $x = 1$  (double)  $x = 3$

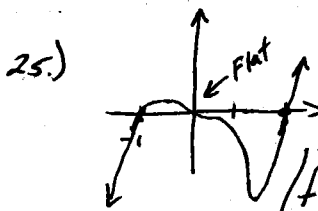
$$h(x) = K(x+2)(x+1)(x-1)^2(x-3)$$

$$-1 = K(2+2)(2+1)(2-1)^2(2-3)$$

$$-1 = -12K$$

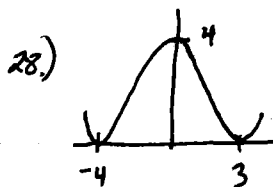
$$\frac{1}{12} = K$$

$$h(x) = \frac{1}{12}(x+2)(x+1)(x-1)^2(x-3)$$



Zeros:  $x = -1$   $x = 0$   $x = 2$   
 Multiple

$$f(x) = Kx^3(x+1)(x-2) \quad K > 0$$



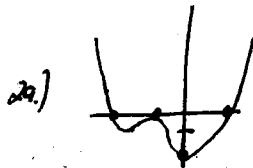
Zeros:  $x = -4$   $x = 3$  (Both Double)

$$f(x) = K(x+4)^2(x-3)^2$$

$$4 = K(0+4)^2(0-3)^2$$

$$4 = 144K$$

$$\frac{1}{36} = K \quad f(x) = \frac{1}{36}(x+4)^2(x-3)^2$$



Zeros:  $x = -2$   $x = -1$  (double)  $x = 1$   
 y-int:  $(0, -2)$

$$h(x) = K(x+2)(x+1)^2(x-1)$$

$$-2 = K(0+2)(0+1)^2(0-1)$$

$$-2 = -2K$$

$$1 = K$$

$$h(x) = (x+2)(x+1)^2(x-1)$$

35.)  $y = (x^2 - 8x + 12)(x-3)$

$$0 = (x-6)(x-2)(x-3)$$

$$x = 6 \quad x = 2 \quad x = 3$$

47) a.)  $f$  has zeros at  $x = -2$   $x = 3$   $x = 5$  and  $y$ -intercept of 4.

$$f(x) = k(x+2)(x-3)(x-5)$$

$$4 = k(0+2)(0-3)(0-5)$$

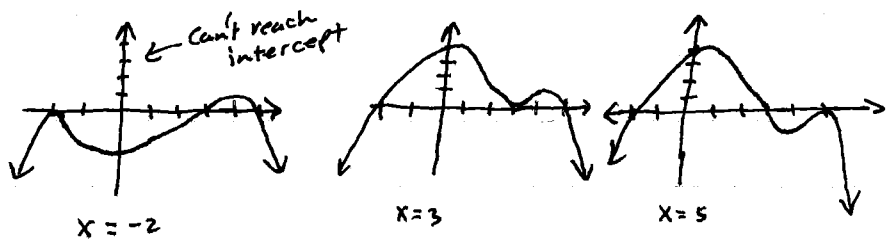
$$4 = 30k$$

$$\frac{2}{15} = k$$

$$f(x) = \frac{2}{15}(x+2)(x-3)(x-5)$$

b.) Same as above, but as  $x \rightarrow \pm\infty, y \rightarrow -\infty$

Could have a double zero at  $x = 3$  or  $x = 5$  but not  $x = -2$ .



So:  $f(x) = k(x+2)(x-3)^2(x-5)$  or  $f(x) = k(x+2)(x-3)(x-5)^2$

$$4 = k(0+2)(0-3)^2(0-5)$$

$$4 = -90k$$

$$-\frac{2}{45} = k$$

$$4 = k(0+2)(0-3)(0-5)^2$$

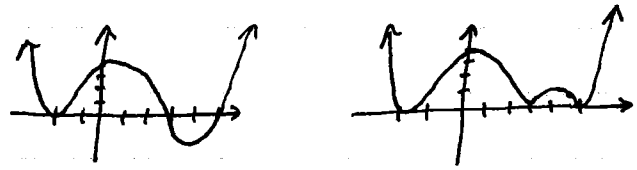
$$4 = 150k$$

$$-\frac{2}{75} = k$$

$$f(x) = -\frac{2}{45}(x+2)(x-3)^2(x-5) \quad \text{or} \quad f(x) = -\frac{2}{75}(x+2)(x-3)(x-5)^2$$

c.) Same as a, but as  $x \rightarrow \pm\infty, y \rightarrow \infty$ .

Must have a double zero at  $x = -2$  to reach  $y$ -intercept of 4. Could have single zeros at  $x = 3$  and  $x = 5$  or double zeros at both:



$$f(x) = k(x+2)^2(x-3)(x-5)$$

$$4 = k(0+2)^2(0-3)(0-5)$$

$$4 = 60k$$

$$\frac{1}{15} = k$$

$$\text{or } f(x) = k(x+2)^2(x-3)^2(x-5)^2$$

$$4 = k(0+2)^2(0-3)^2(0-5)^2$$

$$4 = 900k$$

$$\frac{1}{225} = k$$

$$y = \frac{1}{15}(x+2)^2(x-3)(x-5) \quad \text{or} \quad y = \frac{1}{225}(x+2)^2(x-3)^2(x-5)^2$$